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1991 J. Phys. A: Math. Gen. 24 L949

(http://iopscience.iop.org/0305-4470/24/16/011)

J. Phys. A: Math. Gen. 24 (1991) L949-L954. Printed in the UK

LETTER TO THE EDITOR

Renormalization-group approach to the local Green functions of a family of generalized Fibonacci lattices

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Received 3 April 1991

Abstract. We study the local electronic properties of a family of generalized Fibonacci lattices associated with the sequences which are given by the inflation rule $(A, B) \rightarrow (AB^n, A)$, where n is an arbitrary positive integer greater than one. A unified real-space renormalization-group approach is presented to calculated the local Green function and the local density of states at any given site.

In recent years there has been a growing interest in the theoretical study of the electronic and phonon properties of the one-dimensional (1D) Fibonacci lattice [1-9] and the generalized Fibonacci lattices [10-16]. Kohmoto *et al* [1] introduced a dynamical-systems-theory method to study the Fibonacci lattice and later this method was applied to deal with the generalized Fibonacci lattices [10-14]. Results showed that, for the Fibonacci lattice, the spectrum is a Cantor set [1-6]; while for the generalized Fibonacci lattices more physical properties were found particularly for a family of the generalized Fibonacci lattices corresponding to the sequences S_∞ given by the recursion relation $S_{l+1} = \{S_l, S_{l-1}^n\}$ for $l \geqslant 1$ with $S_0 = \{B\}$ and $S_1 = \{A\}$, in which n is an arbitrary positive integer greater than one [10-16]. This family of lattices are termed the *Bn* chains here for convenience according to the inflation rule $(A, B) \rightarrow (AB^n, A)$.

On the other hand, since the quasiperiodic systems have no translational invariance, every site in the chain has a different environment. So it is important to study the local electronic and phonon properties. For the Fibonacci chain, Ashraff and Stinchcombe [17], Chakrabarti *et al* [18] and Capaz *et al* [19] recently calculated the local Green function (LGF) and the local density of states (LDOS) at a particular site. More recently, an exact real-space renormalization-group scheme was given by Zhong *et al* [20, 21] to obtain the LGF and LDOS at any given site. However, to our knowledge, attention has, to date, mainly concentrated on the Fibonacci chain. Stimulated by the interesting results of the *Bn* chain, we present here a unified RSRG approach to calculate the LGF and the LDOS of the *Bn* chain.

In our study, we emply the following 1D electronic tight-binding Hamiltonian:

$$H = \sum_i |i\rangle \varepsilon_i \langle i| + \sum_{ij} |i\rangle V_{ij} \langle j| \tag{1}$$

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0305-4470/91/160949+06$03.50

where the nearest-neighbour hopping integral V_{ij} takes two kinds of values V_A and V_B represented by letters A and B, respectively; A and B are arranged in the Bn sequence. The site energy ε_i takes one of the four values according to the local environment of site i

$$\varepsilon_i = \begin{cases} \varepsilon_\alpha & \text{if } V_{i-1,i} = V_{i,i+1} = V_A \\ \varepsilon_\beta & \text{if } V_{i-1,i} = V_A \text{ and } V_{i,i+1} = V_B \\ \varepsilon_\gamma & \text{if } V_{i-1,i} = V_B \text{ and } V_{i,i+1} = V_A \\ \varepsilon_\delta & \text{if } V_{i-1,i} = V_{i,i+1} = V_B. \end{cases}$$

The elements of the Green function satisfy the following set of equations

$$(E + i0^+ + \varepsilon_i)G_{ij} = \delta_{ij} + \sum_k V_{ik}G_{kj} \qquad i, j = 0, \pm 1, \pm 2, \ldots. \tag{2}$$

The LDOS at site i is given by

$$\rho_i(E) = -\frac{1}{\pi} \operatorname{Im} G_{ii}(E + i0^+) \tag{3}$$

where Im denotes the imaginary part of a complex.

In order to calculate the LGF at any site in a Bn chain, we introduce $2n+1$ basic renormalization transformations $T_1, T_2, \ldots, T_{2n+1}$. Transformations $T_1, T_2, \ldots T_{n+1}$ are represented by

$$(B^nA^{n+1}, B^nA) \to (A', B')$$

$$(B^{n-1}A^{n+1}B, B^{n-1}AB) \to (A', B'), \ldots, (A^{n+1}B^n, AB^n) \to (A', B')$$

respectively, while the representations of the transformations $T_{n+2}, T_{n+3}, \ldots, T_{2n+1}$ are

$$[A^{n-1}(AB^n)^{n+1}A, A^{n-1}(AB^n)A] \to (A', B')$$

$$[A^{n-2}(AB^n)^{n+1}A^2, A^{n-2}(AB^n)A^2]$$

$$\to (A', B'), \ldots, [(AB^n)^{n+1}A^n, (AB^n)A^n] \to (A', B')$$

respectively. These $2n+1$ basic transformations have the properties that, if we apply them to a Bn chain individually, $2n+1$ new Bn chains are obtained and all the sites of the original chain are transferred to the new chains. The sites of the different new chains are different (see figure 1). It is found that transformations T_1 and T_{n+1} classify

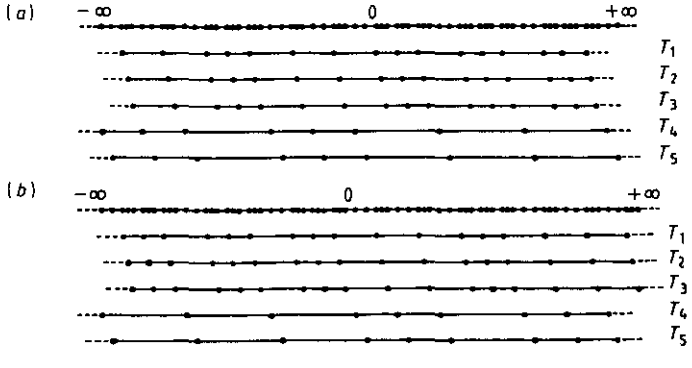

Figure 1. A schematic representation of the five basic transformations for the $B2$ chain. (a) A $B2$ chain with key site (site 0) of type S_β; (b) a $B2$ chain with key site of type S_γ.

the *Bn* chains of different generations into two distinct classes with key sites of different types denoted by type S_β or S_γ, in which each chain has only one key site and no successive generations belong to the same class. The characteristic feature of the key site of type S_β or S_γ is that, through the transformation T_1 or T_{n+1}, this site remains undecimated and the environment of it in the new chain is the same as that in the original one. Therefore the LGF of the key site S_β and S_γ in an infinite *Bn* chain can be obtained by successive iterations of transformation t_1 or T_{n+1}. For the other sites of the *Bn* chain, we can convert them to a key site of a renormalized *Bn* chain by suitable combinations of the $2n+1$ transformations. We first focus our attention on the sites near the key site of type S_β (site 0). It is found that transformations $T_2, T_3, \ldots, T_{n+1}$ make the sites $1, 2, \ldots, n$ become the key sites of type S_β of the new *Bn* chains, respectively, while transformations $T_{n+2}, T_{n+3}, \ldots, T_{2n+1}$ transfer the sites $-n, -(n-1), \ldots, +1$ to the key sites of type S_γ respectively. If site 0 is the key site of type S_γ, transformations $T_1, T_2, \ldots, T_n$ convert the sites $-n, -(n-1), \ldots, -1$ to the key sites of type S_γ and $T_{n+2}, T_{n+3}, \ldots, T_{2n+1}$ make the sites $1, 2, \ldots, n$ become the key sites of type S_β, respectively. From the above statement, we then have the following conclusions: for an infinite *Bn* chain, when we apply the $2n+1$ basic transformations to it individually, $2n+1$ new chains are obtained. We can first identify a special site called the key site by transformation T_1 or T_{n+1}, while the $2n$ sites near the key site become the key sites of the $2n$ new *Bn* chains obtained by other $2n$ transformations. For each new chain, same operations can be taken and then more sites become the key sites. In this way we can convert any given site of the *Bn* chain to a key site in a renormalized *Bn* chain and obtain the corresponding LGF. We present figure 1 for the *B*2 chain to illustrate our RSRG scheme for the *Bn* chain. The five transformations are T_1, T_2, T_3, T_4 and T_5 represented by

$$(BBAAA, BBA) \rightarrow (A', B')$$

$$(BAAAB, BAB) \rightarrow (A', B')$$

$$(AAABB, ABB) \rightarrow (A', B)$$

$$(AABBABBA, AABBA) \rightarrow (A', B)$$

$$(ABBABBAA, ABBAA) \rightarrow (A', B).$$

According to (2) and the RSRG scheme described above, we obtained the recursion relation of transformation T_1 for the *Bn* chain:

$$\varepsilon'_\alpha = \varepsilon_\beta + \frac{1}{X}(V_A Y + V_B Z) \qquad \varepsilon'_\beta = \varepsilon_\beta + \frac{V_A Y}{X} + \frac{V_B P_{n-2}}{P_{n-1}}$$

$$\varepsilon'_\gamma = \varepsilon_\beta + \frac{V_B Z}{X} + \frac{V_A^2 U_{n-1}(g)}{P_{n-1}} \tag{4}$$

$$\varepsilon'_\sigma = \varepsilon_\beta + \frac{1}{P_{n-1}}[V_B P_{n-2} + V_A^2 U_{n-1}(g)]$$

$$V'_A = \frac{V_A V_B}{X} \qquad V'_B = \frac{V_A V_B}{P_{n-1}}$$

where

$$\begin{aligned} &g=(E-\varepsilon_\delta)/V_B,\ h=(E-\varepsilon_\alpha)/V_A \\ &P_n=(E-\varepsilon_\gamma)U_n(g)-V_BU_{n-1}(g) \\ &Q_n=(E-\varepsilon_\gamma)U_n(h)-V_AU_{n-1}(h) \\ &X=U_{n-1}(g)Q_n-V_BU_{n-2}(g)U_n(h) \\ &Y=U_{n-1}(g)Q_{n-1}-V_BU_{n-2}(g)U_{n-1}(h) \\ &Z=U_n(h)P_{n-2}-V_AU_{n-2}(g)U_{n-1}(h) \end{aligned} \tag{5}$$

and $U_n(g)$ is the nth Chebyshev polynomial of the second kind [12, 13, 22] which obeys the recursion relation

$$U_{n+1}(g)=gU_n(g)-U_{n-1}(g) \qquad n\geqslant 1 \tag{6}$$

with initial conditions $U_{-1}(g)=0$ and $U_0(g)=1$. Using the recursion relations, we can easily calculate the LGF and the LDOS at the key site of an infinite Bn chain. It is not difficult to derive the recursion relations of the other $2n$ transformations. As typical examples, we illustrate the LDOS at the key site of type S_B of the Bn chain on the off-diagonal model in figures 2(a)-(c) for $n=2, 3$, and 4, respectively. For the Fibonacci chain, the spectrum is a Cantor set [1-6] and the LDOS [17-21] has no smooth part. However, figures 2(a)-(c) show that in some regions of the spectrum the LDOS seems to have a roughly constant behaviour. This precise point can be well explained [23]: thus, there are a finite density of molecules containing n B in Bn. Then, there exists exact extended eigenstates which can be built from the Dirichlet states of the $(n-1)$ molecules. Their energies are

$$E_l=2\cos\left(\frac{\pi l}{n}\right) \qquad l=1,\ldots,n-1. \tag{7}$$

One finds the set $\{0\}$ for $n=2$, $\{-1, 1\}$ for $n=3$, and $\{-\sqrt{2}, 0, \sqrt{2}\}$ for $n=4$. These are clearly the energies which correspond to the smoothness of the LDOS shown in figure 2. The reason is the following. One can associate a trace map with the dynamics of transfer matrices. As in the case of the Fibonacci chain, there is an invariant $\mathcal{I}$, and the periodic case corresponds to the value $\mathcal{I}=0$. But contrary to the Fibonacci case, $\mathcal{I}$ does depend on E and becomes 0 precisely for the E_l'. Since the DOS is given by the $\cos^{-1}$ of the trace, it is locally smooth. In fact it is even almost constant since $E=E_l$ corresponds to the energy 0 of the periodic chain where the DOS is stationary. Since the associated state is extended the LDOS behaves in the same way. A similar phenomenon was obtained by Site and Mosseri [24] around a gap closing of any kind of quasiperiodic chain.

In summary, we have presented a unified RSRG scheme to study the local electronic properties of the Bn chain which underlying sequence is given by the inflation rule $(A, B)\rightarrow(AB^n, A)$ as a simple generalization of the Fibonacci sequence. In our scheme, $2n+1$ basic renormalization transformations $T_1, T_2, \ldots, T_{2n+1}$ are introduced. For an infinite Bn chain, we can identify a special site called the key site. The LGF and the LDOS of the key site can be calculated by successive iterations of transformation T_1 or T_{n+1} according to the type of the key site. Any other site can be transferred to a key site of a renormalized Bn chain by suitable combinations of the $2n+1$ basic transformations.

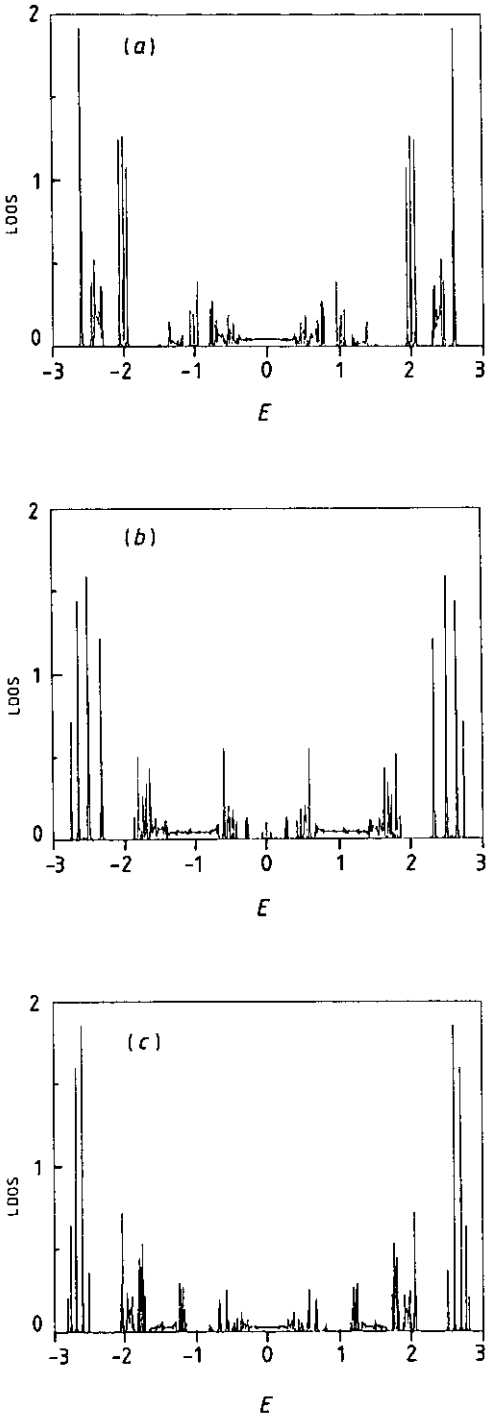

Figure 2. The LDOS (arbitrary units) at the key site of type S_β for the Bn chain, in which $V_A = 1$, $V_B = 1.5$ and $\varepsilon_\alpha = \varepsilon_\beta = \varepsilon_\gamma = \varepsilon_\delta = 0$. (*a*) $B2$ chain; (*b*) $B3$ chain; (*c*) $B4$ chain.

This work has been supported by the National Natural Science Foundation of China and in part by CCAST (World Laboratory).

References

[1] Kohmoto M, Kadanoff L P and Tang C 1983 *Phys. Rev. Lett.* **50** 1870
[2] Ostlund S, Pandit R, Rand D, Schlinhuber H J and Siggia E D 1983 *Phys. Rev. Lett.* **50** 1783
[3] Kohmoto M, Sutherland B and Tang C 1987 *Phys. Rev.* B **35** 1020
[4] Lu J P, Odagaki T and Birman J L 1986 *Phys. Rev.* B **33** 4809
[5] Luck J M and Petritis D 1986 *J. Stat. Phys.* **42** 289
[6] Niu Q and Nori F 1986 *Phys. Rev. Lett.* **57** 2057
[7] Sire C and Mosseri R 1989 *J. Physique* **50** 3447
[8] Delyon F and Petritis D 1986 *Commun. Math. Phys.* **103** 441
[9] Bombieri E and Taylor J E *J. Physique Colloq.* **47** C3-19
[10] Gumbs G and Ali M K 1988 *Phys. Rev. Lett.* **60** 1081
[11] Holzer M 1989 *Phys. Rev.* B **38** 1709
[12] You J Q, Yang Q B and Yan J R 1990 *Phys. Rev.* B **41** 7491
[13] Kŏlár M and Ali M K 1990 *Phys. Rev.* B **41** 7108
[14] Severin M and Riklund R 1989 *Phys. Rev.* B **39** 10362
[15] Severin M, Dulea M and Riklund R 1989 *J. Phys: Condens. Matter.* **1** 8851
[16] Severin M and Riklund R 1989 *J. Phys: Condens. Matter* **1** 5607
[17] Ashraff J A and Stinchcombe R B 1988 *Phys. Rev.* B **37** 5723
[18] Chakrabarti A, Karmakar S N and Moitra R K 1989 *Phys. Rev.* B **39** 9730
[19] Capaz R B, Koiller B and de Queiroz S L A 1990 *Phys. Rev.* B **42** 6402
[20] Zhong J X, You J Q, Yan J R and Yan X H 1991 *Phys. Rev.* B in press
[21] Zhong J X, Yan J R, Yan X H and You J Q 1991 *J. Phys: Condens. Matter* **3** 5685
[22] Kalugin P A, Pitayev A Y and Levitov L S 1986 *Sov. Phys.-JETP* **64** 410
[23] Sire C 1991 private communication
[24] Sire C and Mosseri R 1990 *J. Physique* **51** 1569